Tutorial 2
Advanced Graph Theory

7^{th} August 2014

1. Prove that every n-vertex graph with atleast n edges contains a cycle.
2. If every vertex of a loopless graph G has degree 3, then prove that G has a cycle of even length.
3. Suppose that G is a graph and D is an orientation of G that is strongly connected. Prove that if G has an odd cycle, then D has an odd cycle.

4. Which of the following are graphic sequences?
    - 4.1 (5,5,4,3,2,2,2,1)
    - 4.2 (5,5,4,4,2,2,1,1)
    - 4.3 (5,5,5,3,2,2,1,1)
    - 4.4 (5,5,5,4,2,1,1,1)
5. Let v be a vertex in a connected graph G. Prove that there exists a spanning tree T of G such that the distance of every vertex from v is the same in G and in T.
6. Let T be a tree of order n. Prove that T is isomorphic to a subgraph of C'_{n+2} (complement of C_{n+2}).

7. Prove or disprove: Every tree has atmost one perfect matching.
8. Prove that every maximal matching in a graph G has size at least half the size of a maximum matching.
9. Consider a bipartite graph G with partite sets X and Y. For any subset of vertices S, let $N(S)$ denote the set of vertices such that there is an edge from vertex in S to some vertex in $N(S)$. Show that if $|N(S)| \geq |S|d$ for every subset S of X and some fixed positive integer d, then G has a matching of size, $|X| - d$.